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Title: **Continuum limits of Nonlocal p-Laplacian evolution and variational problems on graphs**

The non-local p-Laplacian evolution equation and variational regularization, governed by a given kernel, have applications in various areas of science and engineering. In particular, there are modern tools for massive data processing (including signals, images, geometry), and machine learning tasks such as classification. In practice, however, these models are implemented in discrete form (in space and time, or in space for variational regularization) as a numerical approximation to a continuous problem, where the kernel is replaced by an adjacency matrix of graph. Yet few results on the consistency of these discretizations are available. In particular it is largely open to determine when do the solutions of either the evolution equation or the variational problem of graph-based tasks converge (in an appropriate topology), as the number of vertices increases, to a well-defined object in the continuum setting, and if yes, at which rate. In this work, we lay the foundations to address these questions.

We give a rigorous interpretation to the continuous limit of the discrete non-local p-Laplacian evolution and variational problems on graphs. We consider a sequence of (deterministic) graphs converging to a so-called limit object known as the graphon. We prove that the solutions of the sequence of discrete problems converge to the solution of the continuous problem governed by the graphon, when the number of graph vertices grows to infinity. Along the way, we provide a consistency/error estimate which is of interest in its own. In turn, this allows to establish the convergence rates for different graph models. In particular, we point out the role of the graphon geometry/regularity. For random graph sequences, we deliver non-asymptotic convergence rates in probability and exhibit the different regimes depending on p and the regularity of the graphon and the initial condition.