Title: Multiscale decomposition of functions in Metric Random Walk Spaces

Our aim is to study the \((BV, L^2)\)-decomposition and the \((BV, L^1)\)-decomposition in the general framework of metric random walk space (MRWS). For instance, the \((BV, L^2)\)-decomposition in the MRWS \([X, d, m]\) with invariant measure \(\nu\) reads as

\[
\min \left\{ \frac{1}{2} \int_X \int_X |u(y) - u(x)| d\mu(x) d\nu(x) + \frac{\lambda}{2} \int_X |u(x) - f(x)| d\nu(x) : u \in L^1(X, \nu) \right\},
\]

which has as particular case the ROF-model in a weighted graph \(G = (V(G), E(G), W(G))\):

\[
\min \left\{ \frac{1}{2} \sum_{x \in V(G)} \sum_{y \in V(G)} |u(y) - u(x)| w_{xy} + \frac{\lambda}{2} \sum_{x \in V(G)} |u(x) - f(x)|^2 \sum_{y : y \sim x} w_{xy} : u \in L^2(G) \right\},
\]

and also the nonlocal ROF-model

\[
\min \left\{ \int_{\Omega \times \Omega} J(x - y)|u(x) - u(y)| dx dy + \frac{\lambda}{2} \|u - f\|_{L^2}^2 : u \in L^2(\Omega) \right\}.
\]

Furthermore, we introduce the concepts of Cheeger and calibrable sets in MRWS and characterize calibrability by using the 1-Laplacian operator. In connection with the Cheeger cut problem we study the eigenvalue problem whereby we give a method to solve the optimal Cheeger cut problem.

Joint work with M. Solera and J. Toledo