International Workshop on Nonlocal Models, PDEs and Applications

NMPDESA2019

May 13-14, 2019

Caen, Normandy, FRANCE

WORKSHOP PROGRAM

RÉGION NORMANDIE

European Commission

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Preface

Recently, there has a wave of interest in developing continuous or discrete nonlocal models and PDEs for a variety of problems arising in physics, biology, social sciences, image processing and machine learning.

Whereas local models based on classical differential operators capture the properties at point for a given function, nonlocal models based on integral operators of an appropriate structure can be used to model phenomena characterized by singular or discontinuous behaviors. They can also be used to approximate local models as the width of the interaction kernel shrinks to zero on the space of sufficiently smooth functions.

This workshop on Nonlocal Models, PDEs and Applications 2019 will focus on recent developments in the area of nonlocal models, PDEs and their various applications. It will bring together internationally recognized experts for young researchers and students are encouraged to present their work during poster sessions.

This Workshop is funded by the **MoNoMaD** project : **Mo**dèles **No**n locaux et **Ma**sse de **D**onnées : de la théorie, l'aide à la décision en Imagerie Médicale à la valorisation du Patrimoine 3D Normand (Réseaux d'Intérêts Normands - **RIN**) and the European Union's Horizon 2020 research and innovation programme under the Marie Sk lodowska-Curie grant agreement No 777826 (**NoMADS**).

The conference organizers would like to express their cordial gratitude to the Invited Speakers for having accepted to participate to Workshop on Nonlocal Models, PDEs and Applications (Caen) and for their state-of-the-art plenary presentations.

The conference organizers would like to thank of course all our partners whether it is the Regional Council of Normandy, the European Union's, the Normandy Interest Network (RIN), The CNRS research federation Normastic.

We would also like to thank the members of the local committee for their advice and help.

Finally, we were very pleased to welcome all the participants to this Workshop and we look forward to meet you at the next edition of NMPDESA Workshop.

A. ELMOATAZ

Z. LAKHDARI

M. LOUAKED

Organizing Committe

A. ELMOATAZ
Laboratoire GREYC
Université de Caen-Normandie

Z. LAKHDARI
Laboratoire GREYC
Université de Caen-Normandie

M. LOUAKED
LMNO (Laboratoire de Mathématiques Nicolas Oresme)
Université de Caen-Normandie

Topics

- Local and Nonlocal Partial Differential Equations
- PDES and Applications
- Partial Differential Equations on Graphs and Networks
- Nonlocal techniques in image processing and machine learning
- Numerical Methods for PDE

Registration

Registration is mandatory to participate to this event.

The registration fees for the workshop are free.

For that, just send an email to: npde-vpa2019@sciencesconf.org

by indicating **your Name, Surname, Position, University or Laboratory of membership as well as your Address**.

WORKSHOP PROGRAM

Schedule	May 13, 2019 (AM)	Location
08h00-10h00	Registration	Hall
09h00-9h10	Opening Ceremony	S3-057 Amphitheater
09h10-10h05	Keynote n°1 : **Nikos KATZOURAKIS** University of Reading, United Kingdom	S3-057 Amphitheater
10h05-10h20	Coffee Break	Hall
10h20-11h15	Keynote n°2 : **Jalal FADILI** ENSICAEN, Université de Normandie, FRANCE	S3-057 Amphitheater
11h15-12h10	Keynote n°3 : **Nicolas FORCADEL** INSA de Rouen Normandie, FRANCE	S3-057 Amphitheater

Schedule	May 13, 2019 (PM)	Location
13h45-14h40	Keynote n°4 : **Antonin CHAMBOLLE** CMAP, École Polytechnique, Palaiseau FRANCE	S3-057 Amphitheater
14h40-15h35	Keynote n°5 : **Matthew THORPE** University of Cambridge, United Kingdom	S3-057 Amphitheater
15h35-16h10	**Poster Session** Coffee Break	Hall
16h10-17h05	Keynote n°6 : **Julian TOLEDO** Université de Valence, ESPAGNE	S3-057 Amphitheater
17h05-18h00	Keynote n°7 : **Abdelkader SAÏDI** Université de Strasbourg, FRANCE	S3-057 Amphitheater

Schedule	May 14, 2019 (AM)	Location
08h30-09h25	Keynote n°8 : **José MAZON** Université de Valence, ESPAGNE	S3-057 Amphitheater
09h25-10h20	Keynote n°9 : **Zakaria BELHACHMI** Université de Haute Alsace, FRANCE	S3-057 Amphitheater
10h20-10h35	Coffee Break	Hall
10h35-11h30	Keynote n°10 : **Yves Van GENNIP** **TU Delft, Nederlands**	S3-057 Amphitheater
11h30-12h25	Keynote n°11 : **Noureddine IGBIDA** Université de Limoges, FRANCE	S3-057 Amphitheater

Schedule	May 14, 2019 (PM)	Location
13h45-14h40	Keynote n°12 : **Farid BOZORGNIA** Tecnico Ulisboa University, PORTUGAL	S3-057 Amphitheater
14h40-15h35	Keynote n°13 : **Fahd KARAMI** Université Cady Ayyad, MOROCCO	S3-057 Amphitheater
15h35-15h50	Coffee Break	Hall

Keynotes Abstracts

Keynote n°1 : **Nikos KATZOURAKIS**
University of Reading, United Kingdom

Title : **Inverse optical tomography through PDE constrained optimisation in L^∞**

Fluorescent Optical Tomography (FOT) is a new bio-medical imaging method with wider industrial applications. It is currently intensely researched since it is very precise and with no side effects for humans, as it uses non-ionising red and infrared light. Mathematically, FOT can be modelled as an inverse parameter identification problem, associated with a coupled elliptic system with Robin boundary conditions. In this talk I will explain how one case utilise novel methods of Calculus of Variations in L^∞ to lay the mathematical foundations of FOT which is posed as a PDE-constrained minimisation problem in L^p and L^∞.

N. Katzourakis, Department of Mathematics and Statistics, University of Reading, Whiteknights, PO Box 220, Reading RG6 6AX, United Kingdom
E-mail address: n.katzourakis@reading.ac.uk

Keynote n°2 : **Jalal FADILI**
ENSICAEN, Université de Normandie, FRANCE

Title: **Continuum limits of Nonlocal p-Laplacian evolution and variational problems on graphs**

The non-local p-Laplacian evolution equation and variational regularization, governed by a given kernel, have applications in various areas of science and engineering. In particular, there are modern tools for massive data processing (including signals, images, geometry), and machine learning tasks such as classification. In practice, however, these models are implemented in discrete form (in space and time, or in space for variational regularization) as a numerical approximation to a continuous problem, where the kernel is replaced by an adjacency matrix of graph. Yet few results on the consistency of these discretizations are available. In particular it is largely open to determine when do the solutions of either the evolution equation or the variational problem of graph-based tasks converge (in an appropriate topology), as the number of vertices increases, to a well-defined object in the continuum setting, and if yes, at which rate. In this work, we lay the foundations to address these questions.
We give a rigorous interpretation to the continuous limit of the discrete non-local p-Laplacian evolution and variational problems on graphs. We consider a sequence of (deterministic) graphs converging to a so-called limit object known as the graphon. We prove that the solutions of the sequence of discrete problems converge to the solution of the continuous problem governed by the graphon, when the number of graph vertices grows to infinity. Along the way, we provide a consistency/error estimate which is of interest in its own. In turn, this allows to establish the convergence rates for different graph models. In particular, we point out the role of the graphon geometry/regularity. For random graph sequences, we deliver non-asymptotic convergence rates in probability and exhibit the different regimes depending on p and the regularity of the graphon and the initial condition.

Keynote n°3 : **Nicolas FORCADEL**
INSA de Rouen Normandie, FRANCE

Title: **Justification of macroscopic traffic flow model by specified homogenization of microscopic models.**

The goal of this talk is to present and to justify Hamilton-Jacobi formulation for macroscopic traffic flow model. The idea is to show how it is possible to deduce macroscopic models of traffic flow from microscopic ones. The main advantage of microscopic models (in which we describe the dynamics of each vehicle in an individual way) is that one can easily distinguish each vehicle and then associate different attributes (like maximal velocity, maximal acceleration...) to each vehicle. It is also possible to describe microscopic phenomena like red lights, slowdown or change of the maximal velocity. The main drawback is for numerical simulations where we have to treat a large number of data, which can be very expensive for example if we want to simulate the traffic at the scale of a town.

On the contrary, macroscopic models consist in describing the collective behaviour of the vehicles for example by giving an evolution law on the density of vehicles. The oldest macroscopic model is the LWR model (Lighthill, Whitham [LW55], Richards [R56]), which dates back to 1955 and is inspired by the laws of fluid dynamics. More recently, some macroscopic models propose to describe the flow of vehicles in terms of the averaged spacing between the vehicles (in some sense, the inverse of the density, see the works of Leclercq, Laval and Chevallier [LLC07]). The main advantage of these macroscopic models is that it is possible to make numerical simulations on large portion of road. On the other side, it is more complicated to describe microscopic phenomena or attributes.

Generally speaking, microscopic models are considered more justifiable because the behaviour of every vehicle can be described with high precision and it is immediately clear which kind of interactions are considered. On the contrary, macroscopic models are based on assumptions that are hardly correct or at least verifiable. As a consequence, it is often desirable establishing a connection between microscopic and macroscopic models so to justify and validate the latter on the basis of the verifiable modelling assumptions of the former.

The goal of this talk is to show how to pass from microscopic models to macroscopic ones. As we will explain, this problem can be seen as a homogenization result on a non-local Hamilton-Jacobi equation. More precisely, at the microscopic scale, we will consider a first order model of the type « follow the leader », i.e., the velocity of a vehicle depends only on the distance with the one in front of it and we will consider a local perturbation located at the origin which make slow down the vehicles. At the macroscopic scale, we attend to recover an Hamilton-Jacobi equation on the right and on the left of the origin and a condition of junction at the origin (as studied in the work of Imbert and Monneau [IM15]). This junction condition allows us to see the influence of the microscopic perturbation at the macroscopic scale. We will also consider the case of a simple junction, i.e., one road that separates in several roads.

Keynote n°4 : **Antonin CHAMBOLLE**
CMAP, Ecole Polytechnique, Palaiseau

Title: **Discretizations of the total variation**

We will discuss the order of approximation of different ways, based on finite differences or finite elements, to approximate the total variation of an image for minimization problems.

Keynote n°5 : **Matthew THORPE**
University of Cambridge, United Kingdom

Title: **Continuum Limits of Semi-Supervised Learning on Graphs**

Given a data set $X_n=\{x_i\}_{i=1}^n$ and a subset of training labels $\{y_i\}_{i\in Z_n}$ where $Z_n \subset \{1,...,n\}$ the goal of semi-supervised is to infer labels on the unlabelled data points $\{x_i\}_{i\not\in Z_n}$. In this talk we use a random geometric graph model with connection radius ϵ_n. The framework is to consider objective functionals which reward the regularity of the estimated labels and impose or reward the agreement with the training data, more specifically we will consider discrete p-Laplacian regularization.

The talk concerns the asymptotic behaviour in the limit where the number of unlabelled points increases while the number of training labels becomes asymptotically small. The results are to uncover a delicate interplay between the regularizing nature of the functionals considered and the nonlocality inherent to the graph constructions. To establish asymptotic consistency we make use of a discrete-to-continuum topology that is based on optimal transport and variational methods such as Γ-convergence. I will give almost optimal ranges on the scaling of ϵ_n for asymptotic consistency to hold.

This is joint work with Jeff Calder (Minnesota) and Dejan Slepcev (CMU).

Keynote n°6 : **Julian TOLEDO**
Université de Valence, ESPAGNE

Title: **The total variation ow in metric random walk spaces**

Our aim is to study the total variation flow in metric random walk spaces, which include locally finite weighted connected graphs. We first introduce the concepts of total variation, perimeter and mean curvature in the ambient of metric random walk spaces. We establish some inequalities of Poincaré type. Then we study existence and uniqueness of solution of the total variation flow, and its asymptotic behaviour.

Joint work with J. M. Mazón and M. Solera

Keynote n°7 : **Abdelkader SAÏDI**
Université de Strasbourg, FRANCE

Title: **Numerical Approximation and Control Problem for Water Treatment System**

The protection of the marine and coastal environment became, these last years, a major issue. The reduction of pollutants coming from sewage discharges is one of the challenges in this context. Coastal pollution is generally controlled by treating contaminants at source or at sewage farms by wastewater treatment methods in order to reduce their concentration. Many investigations show that the optimal control theory allows the design of a wastewater treatment system to control marine pollution in any open area of shallow water. The aim of the work presented in this talk is to provide a method, based on the control theory, that enable to maintain the concentration of pollutants close or under a standard value. Both theoretical and numerical issues are discussed. We consider the evolution of waste water concentration model given by a parabolic equation with a pointwise source term modeling the action of the outlets. To improve the water quality, we introduce a control action at the points source by minimizing the deviation of fecal coliformdistribution from standard value. We firstly, give an existence and uniqueness result of the solution of the pollutant transport problem. We present the control problem and explain how to compute the optimal action by introducing an adjoint state. For numerical reasons, we regularize the initial problem. We study then the regularized system and show the convergence of the regular solutions towards that of the initial problem. The approximation of the transport of biological systems is done by an hybrid numerical procedure combining a particle method and a finite difference technique. The control problem is treated by a minimizing algorithm. The gradient of the cost function is evaluated by adjoint techniques. A gradient type method is chosen as an iterative solution of the discrete control problem. In the last part, we present numerical results that show the efficiency of the hybrid scheme associated to a control algorithm.

Keynote n°8 : José MAZON

Université de Valence, ESPAGNE

Title: **Multiscale decomposition of functions in Metric Random Walk Spaces**

Our aim is to study the (BV, L^2)-decomposition and the (BV, L^1)-decomposition in the general framework of metric random walk space (MRWS) For instance, the (BV, L^2)-decomposition in the MRWS $[X, d, m]$ with invariant measure ν reads as

$$\min \left\{ \frac{1}{2} \int_X \int_X |u(y) - u(x)| dm_x(y) d\nu(x) + \frac{\lambda}{2} \int_X |u(x) - f(x)| d\nu(x) : \ u \in L^1(X, \nu) \right\},$$

which has as particular case the ROF-model in a weighted graph $G = (V(G), E(G), W(G))$:

$$\min \left\{ \frac{1}{2} \sum_{x \in V(G)} \sum_{y \in V(G)} |u(y) - u(x)| w_{xy} + \frac{\lambda}{2} \sum_{x \in V(G)} |u(x) - f(x)|^2 \sum_{y \sim x} w_{xy} : \ u \in L^2(G) \right\},$$

and also the nonlocal ROF-model

$$\min \left\{ \int_{\Omega \times \Omega} J(x-y) |u(x) - u(y)| dx dy + \frac{\lambda}{2} \|u - f\|_2^2 \ : \ u \in L^2(\Omega) \right\}.$$

Furthermore, we introduce the concepts of Cheeger and calibrable sets in MRWS and characterize calibrability by using the 1-Laplacian operator. In connection with the Cheeger cut problem we study the eigenvalue problem whereby we give a method to solve the optimal Cheeger cut problem.

Joint work with M. Solera and J. Toledo

Keynote n°9 : Zakaria BELHACHMI

Université de Haute Alsace, FRANCE

Title: **Modelling with high order PDEs for Geometry inpainting**

We consider a class of nonstandard high-order PDEs models, based on the p(·)-Laplace operator with variable exponents, for image denoising and image inpainting problems. These models are obtained from the minimization of a family of anisotropic energies that range between the so called $TV{-}H^{-1}$ and the biharmonic models. The PDEs system consists of quasi-linear equations that we solve with a fixed point iterative method and we prove the convergence of the iterative process. To solve the problems, we consider an algorithm which includes a practical and efficient adaptive strategy for the choice of the exponent $1<p\leq 2$, which allows us to fit to the multi-scale nature of the images. We present several numerical examples to test our approach and to make some comparisons with existing methods.

Keynote n°10 : Yves Van GENNIP
TU Delft, Nederlands

Title: **Deep limits of residual neural networks**

In their study of the continuum limits of variational problems on graphs, Dejan Slepcev and Nicolas Garcia Trillos introduced an elegant method to identify discrete graph-based functions with continuum functions through an optimal transport construction. Since its introduction this method has proven applicable to many graph-to-continuum limiting problems.

In this talk we will see a new setting in which this method has been successfully applied: the deep layer limit of neural networks. In particular, we will show that the training of a residual neural network can be formulated as a constrained discrete variational problem, whose deep layer limit (i.e. #layers --> infinity) is given by a continuum variational problem constrained by an ordinary differential equation.

This is joint work with Matthew Thorpe

Keynote n°11 : Noureddine IGBIDA
Université de Limoges, FRANCE

Title: **On the Eikonal equation and some related problems**

The aim of the talk is to give an overview concerning the connection between the Eikonal equation, the sandpile and the optimal mass transport problem. We'll discuss the continuous as well as the discrete and non local approaches for these problems

Keynote n°12 : Farid BOZORGNIA
Tecnico Ulisboa University, PORTUGAL

Title: **Eigenvalues of p-Laplace and Infinity Laplace operator**

In this talk, under special symmetry assumption on the domain, we approximate the second eigenfunction and the second eigenvalue of p-Laplace operator by a new idea.

Next, we discuss the limiting problem as p tends to infinity. We present an alternative formulation of the infinity Laplace eigenfunction problem, which can be easily used to compute the first and the second eigenvalues and the corresponding eigenfunctions.

Keynote n°13 : **Fahd KARAMI**
Université Cady Ayyad, MOROCCO

Title: **Study of some nonlinear and nonlocal reaction-diffusion system and application for image restoration.**

In this work, we will present a class of nonlocal reaction-diffusion systems applied to the restoration of degraded images. These models are obtained by decomposing the image into a cartoon part and a texture part.

We use a nonlinear regularization of the operator 1- Laplacian, the latter will be coupled with a non-local Fractional distribution. We study the well-posed nature of the proposed system and will also present numerical simulations.

joint work with A. Atlas O.Oubbih et D. Meskine

Venue

The workshop will be held at the UFR Sciences of the University of Caen Basse-Normandie on Campus 2. The Campus 2 of Caen also known as "Campus Côte de Nacre" is one of the campuses of the agglomeration Caen and is located north of Caen.

The workshop will take place in the Science 3 building of Campus 2.

- From the train station, the most convenient (and cheaper) way to access Campus 2 is by bus: Lines 7 and 10, stop "Caen-Campus 2, shopping center". Line 14, stop "marshal June".

- By car: follow the northern ring road and take the exit n ° 5, direction Douvres-la-Délivrande.

The town is easily accessible by car. Paris is 220 km from Caen (autoroute de Normandie – A13).